Artifacts due to Conformal Deformations in Electrical Impedance Tomography

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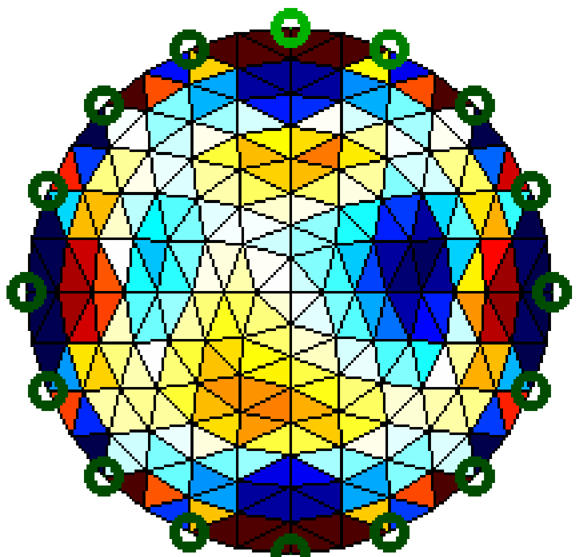
Boundary Movement



Uncorrected

For difference EIT, errors in the boundary cause significant artifacts.

With chest EIT, breathing results in continuous changes in the boundary shape.

(Boyle, *et al* 2008 ”Evaluating Deformation Corrections in Electrical Impedance Tomography”, EIT Conference 2008)

Anisotropic Changes

- Some boundary changes, upon reconstruction, result in anisotropic conductivities:
  - theoretically, for an infinite number of electrodes, non-conformal changes in boundary shapes and electrode locations can be uniquely determined (Lionheart,1998);
  - in some cases, conductivity and shape changes can be recovered using a combined image reconstruction model of both conductivity and shape changes (Soleimani et al, 2006).

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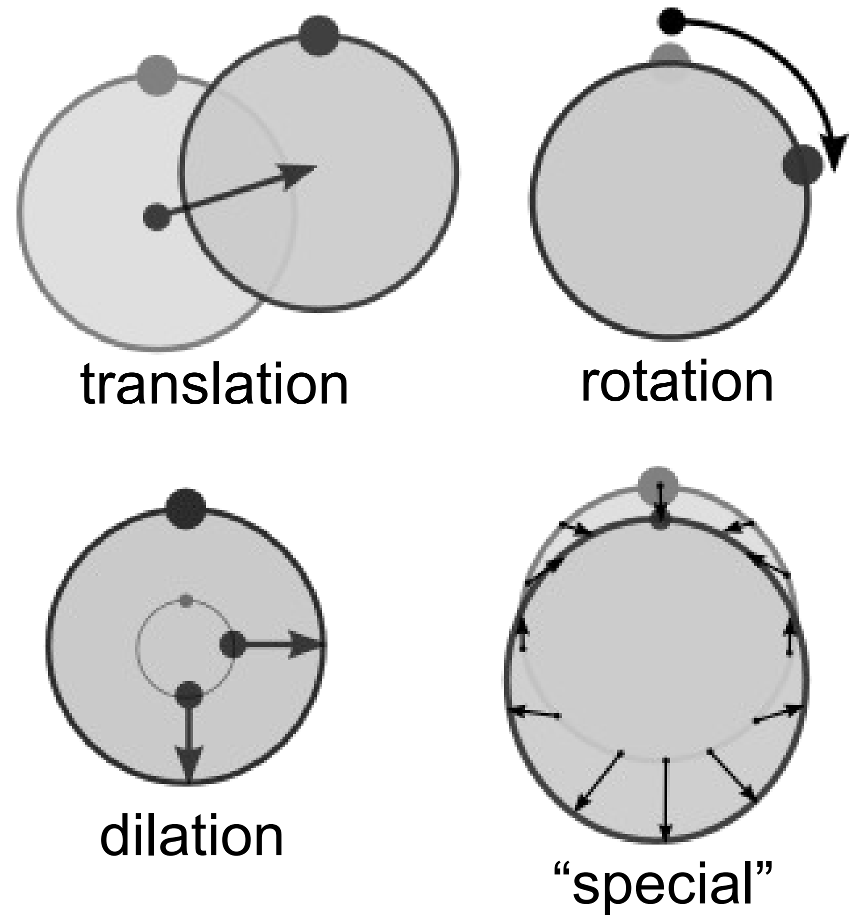
Conformal Deformations
(two dimensions)

- A deformation that locally preserves the angles between vectors.
- Four types:
  - translation,
  - rotation,
  - dilation, and
  - inversion/reflection.



Examples

Möbius $$z \to \frac{az+b}{cz+d}, ad-bc \neq 0$$

Our “special” example $$z \to z + a z^2$$ $=z(1+az)$

where $$z = x_1 + i x_2 \to (x_1 + X_1) + i(x_2 + X_2)$$

Conformal Deformations

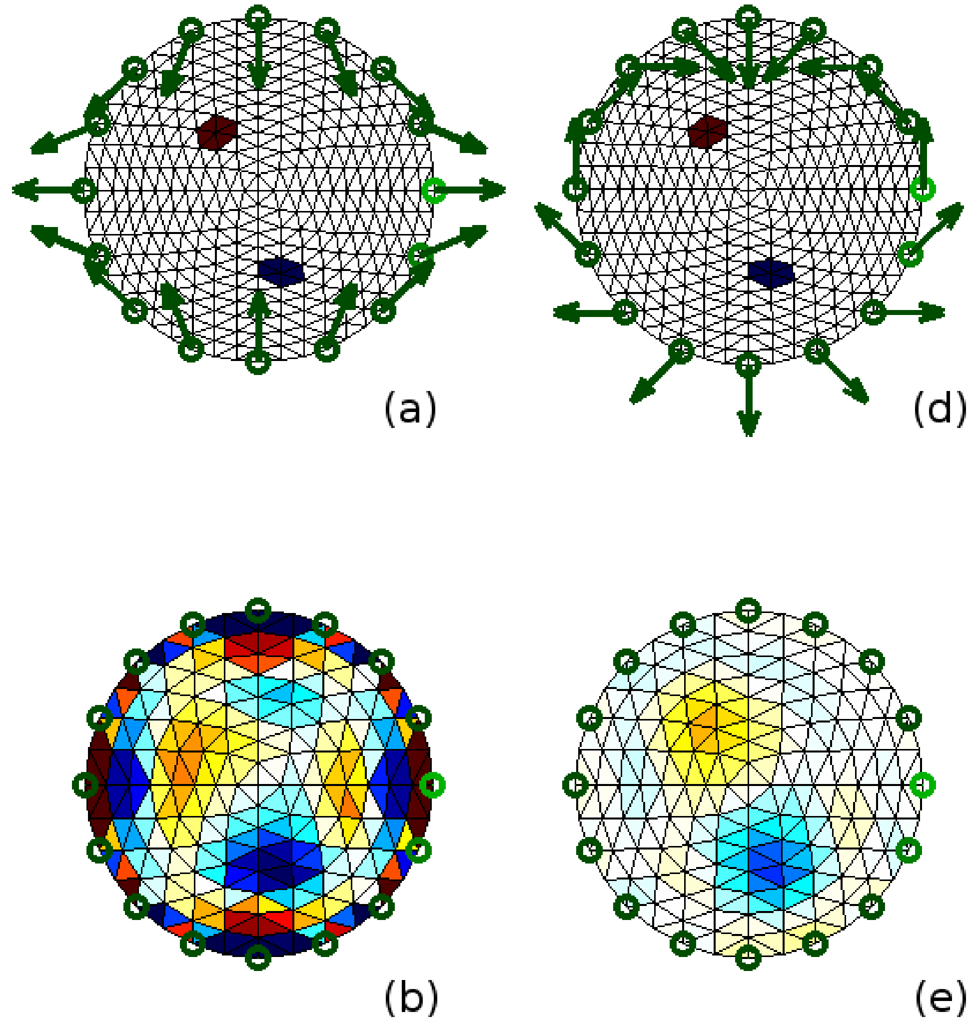
- Conformal deformations
  (and only conformal deformations)
  do *NOT* result in anisotropic conductivity artifacts since they have locally preserved the angles through the deformation.

Examples

Non-Conformal | Conformal

source

without correction



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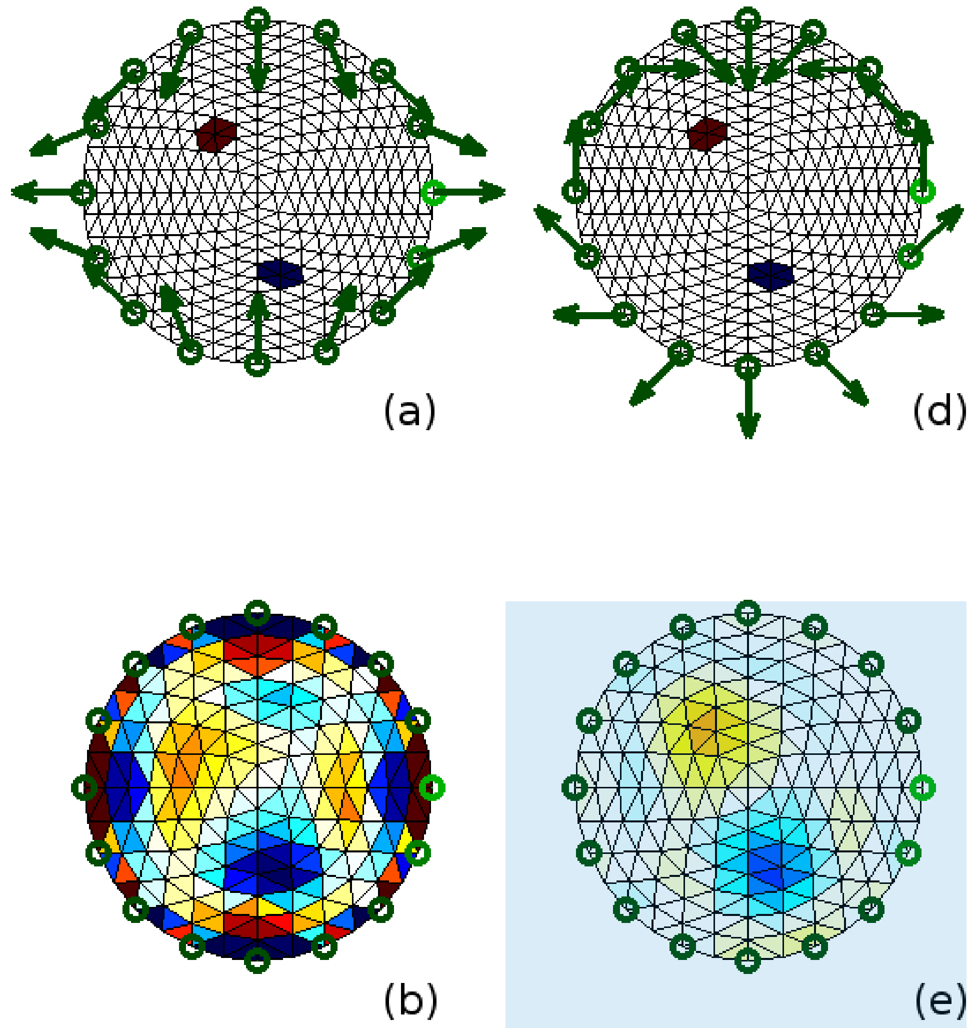

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In EIT

governing equation

$$\nabla \cdot \sigma \nabla \Phi = \begin{cases} 0 & \text{inside} \\ J_n & \text{on the boundary} \end{cases}$$

for a conformal deformation the conductivities match before and after:

$$\nabla \cdot \sigma_c \nabla \Phi_c(x_1, x_2) = \nabla \cdot \sigma_m \nabla \Phi_m(x_1 + X_1, x_2 + X_2)$$

where ***c*** is for conductivity change and ***m*** is for conformal motion

A Bit of Math...

$$\begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial(x_1+X_1)}{\partial x_1} & \frac{\partial(x_1+X_1)}{\partial x_2} \\ \frac{\partial(x_2+X_2)}{\partial x_1} & \frac{\partial(x_2+X_2)}{\partial x_2} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial(x_1+X_1)} \\ \frac{\partial}{\partial(x_2+X_2)} \end{bmatrix}$$

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For a Given Conformal Deformation

- Satisfy the Cauchy-Riemann equations:

where $X = X_1 + i\,X_2$

"the motion"

where $x = x_1 + i\,x_2$

"the basis", ie: x and y axis

$$\frac{\partial X_1}{\partial x_1} - \frac{\partial X_2}{\partial x_2} = 0 \qquad \frac{\partial X_1}{\partial x_2} + \frac{\partial X_2}{\partial x_1} = 0$$

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$$\frac{\partial X_1}{\partial x_1} = \frac{\partial X_2}{\partial x_2} = A - 1 \qquad \frac{\partial X_1}{\partial x_2} = -\frac{\partial X_2}{\partial x_1} = B$$

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Substituting and taking the inverse...

$$\begin{bmatrix} \frac{\partial}{\partial(x_1+X_1)} \\ \frac{\partial}{\partial(x_2+X_2)} \end{bmatrix} = \underbrace{\frac{1}{A^2+B^2}\begin{bmatrix} A & -B \\ B & A \end{bmatrix}}_{T} \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix}$$

In EIT

$$\nabla \cdot \sigma \nabla \Phi = \begin{cases} 0 & \text{inside} \\ J_n & \text{on the boundary} \end{cases}$$

$$\nabla \cdot \sigma_c \nabla \Phi_c(x_1, x_2) = \nabla \cdot \sigma_m \nabla \Phi_m(x_1 + X_1, x_2 + X_2)$$

$$\Phi_c(x_1, x_2) = \Phi_m(x_1 + X_1, x_2 + X_2)$$

(given same boundary measurements)

$$\sigma_c = TT^{\mathrm{T}} \sigma_m \text{ where } TT^{\mathrm{T}} = 1/(A^2 + B^2)$$

$$\sigma_c = \frac{1}{A^2 + B^2} \sigma_m$$

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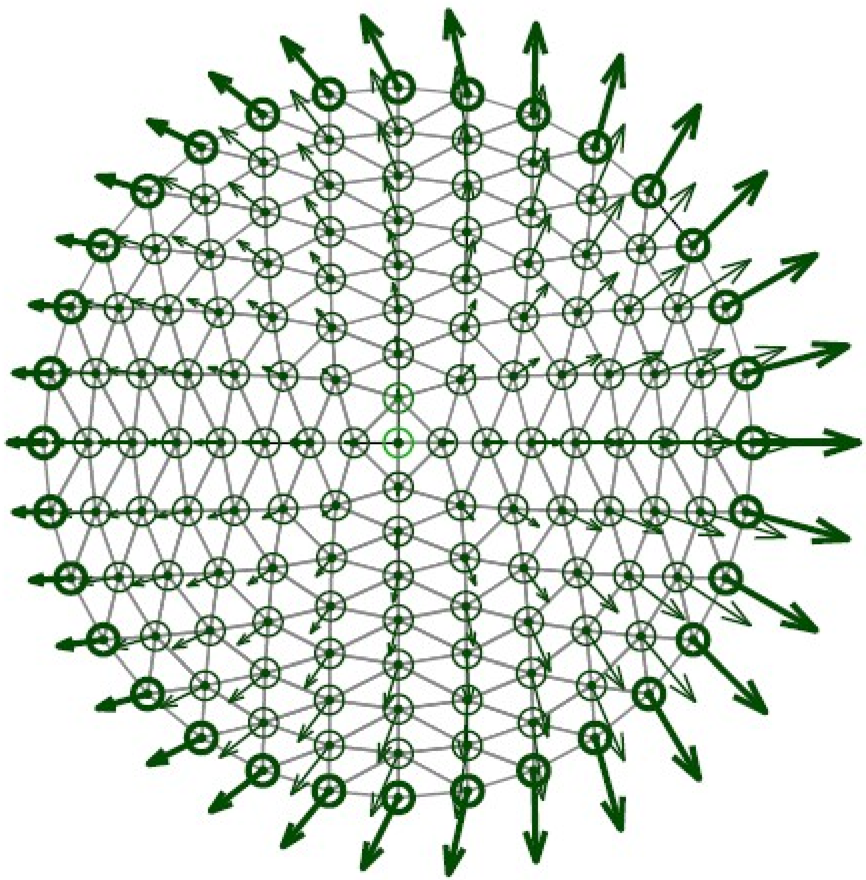
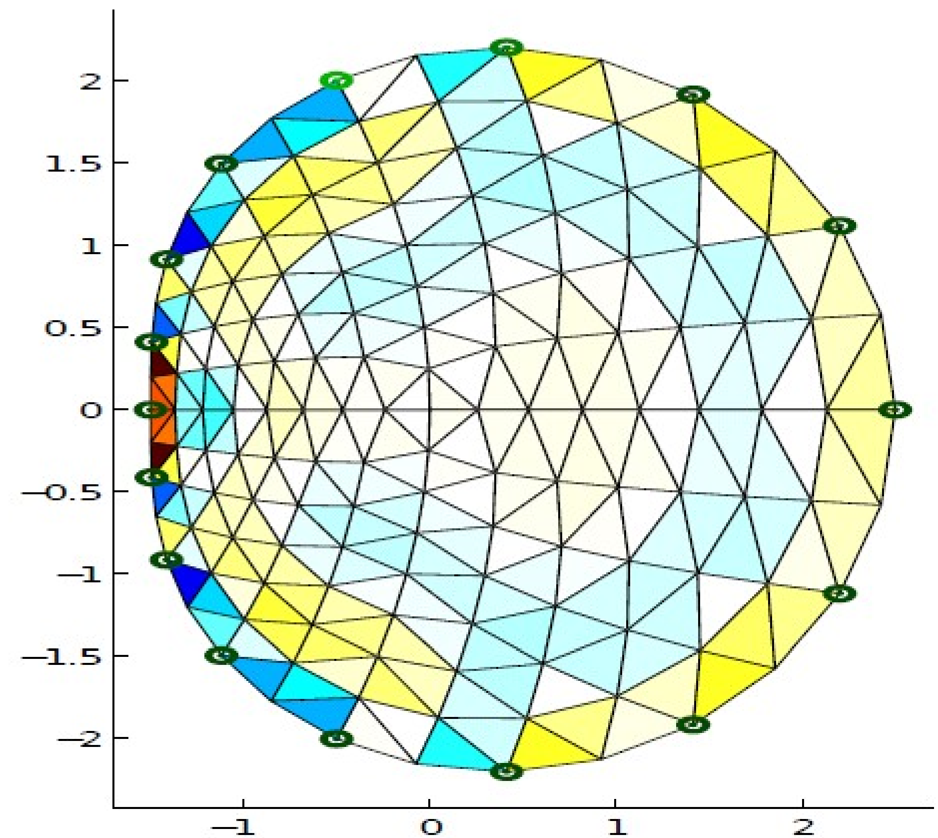


$$z \to z + z^2/2$$



$$\sigma_c = \frac{1}{A^2 + B^2}\sigma_m, \quad \sigma_c = 1$$

Discussion

- Conformal changes don't cause anisotropic conductivities and, thus can't be reconstructed from measurements alone.
- Can apply a better understanding of conformal motions to the reconstruction algorithms:
  - remove the conformal component when analyzing performance, or
  - choose appropriate conformal motion if selecting desired “artifacts” in a difference image

Thank you.

Questions?

Acknowledgement: This work was supported by a grant from NSERC Canada.