# Robust Stimulation and Measurement Patterns in Biomedical EIT

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**Abstract:** Distinguishability criteria incorporating stimulation and measurement patterns as well as an initial conductivity distribution over a specific domain were used in combination with linear programming. The outcomes were stimulation and measurement patterns that maximize the minimum distinguishability to give a robust experimental design. Specific applications for biomedical EIT were demonstrated. The framework is observed to be extensible to arbitrary domains, electrode configurations and geometries.

## 1 Introduction

The quality of images produced in impedance imaging is directly affected by the signal-to-noise ratio of the measurements and electrode connectivity. In some scenarios, image quality can be improved by increasing the stimulation amplitude or reacquiring data. Stimulation amplitude is finite due to equipment properties. Additional electrical safety concerns exist in biomedical applications where high current densities can result in tissue heating and interference with the body's electrical systems. Reacquiring missing data may not be possible due to the cost of a study or the transient nature of the events being observed. In these situations, we would like to utilize stimulation and measurement patterns that provide a robust data set in terms of good distinguishability throughout the imaged region despite the possibility of an a priori unknown set of poorly connected electrodes.

In this work, we develop an approach to determine an optimal stimulation and measurement pattern for a particular arrangement of electrodes and subject. The distinguishability over a set of regions of interest due to a variety of individual stimulation and measurement patterns is determined. Based on the calculated distinguishabilities of these candidate stimulation and measurement patterns, an optimal subset is selected through a linear programming formulation. The resulting selection is constrained so that reconstructed images avoid worst-case artifacts in the event of missing or poor quality data on particular channels. The results were evaluated on two-dimensional simulations of the human thorax showing an acceptable quality of reconstruction using the selected stimulation and measurement patterns.

## 2 Distinguishability

The quality of an EIT system is ultimately determined by its ability to consistently produce impedance images that accurately distinguish between features of interest. Additional factors such as ringing, position errors and reconstruction artifacts degrade the ability of the system to perform this task [1]. Once distinguishability is quantified, systems can be compared knowing that it will perform well if the system provides sufficient distinguishability.

We define distinguishability here as the ability to distinguish between a hypothesis $H_1$ (conductivity change) and the null hypothesis $H_0$ (no conductivity change) within a particular Region of Interest (ROI) according to some measure $m$ [2]. The probability that the null hypothesis is rejected is determined by the $z$-score

$$z = \frac{\hat{m} - m_0}{\text{std}(\hat{m})} \tag{1}$$

where $\hat{m}$ is the maximum likelihood estimate for $m$, the null hypothesis is $m_0$ and std$(m)$ is the standard deviation of $m$. The measure $m$ is the estimated impedance change $\Delta\hat{\sigma}$ for a set of noisy $\mathbf{n}$ difference measurements $\Delta\mathbf{d}$ such that

$$\Delta\mathbf{d} = \mathbf{J}\Delta\hat{\sigma} + \mathbf{n} \tag{2}$$

where $\mathbf{J}$ is the Jacobian for a linearization of the forward EIT problem at the null hypothesis $H_0$. The impedance change for the null hypothesis is zero $m_0 = \Delta\hat{\sigma}_0 = 0$. The maximum likelihood estimate of the conductivity change $\arg\min ||\Delta\mathbf{d} - \mathbf{R}\Delta\sigma|| + P(\cdot)$ for the hypothesis $m$ within an ROI of area $A_R$ is

$$m = \mathbf{\Theta}_R{}^\intercal \Delta\hat{\sigma} = \mathbf{\Theta}_R{}^\intercal \mathbf{R}\Delta\mathbf{d} = A_R \Delta\hat{\sigma}_R \tag{3}$$

where $\mathbf{\Theta}_R$ selects the elements in the ROI weighted by area, $\mathbf{R}_R = \mathbf{\Theta}_R{}^\intercal\mathbf{R}$ is the linearized reconstruction matrix for the measurements. The linearized maximum likelihood reconstruction matrix $\mathbf{R}_R$ is

$$\mathbf{R}_R = (\mathbf{J}_R{}^\intercal \Sigma_n{}^{-1} \mathbf{J}_R)^{-1} \mathbf{J}_R{}^\intercal \Sigma_n{}^{-1} \tag{4}$$

where $\mathbf{J}_R = \mathbf{J}\mathbf{\Theta}_R / A_R = \Delta\mathbf{d}/m$ and $\mathbf{J}$ is the Jacobian of the conductivity change $[\mathbf{J}]_{i,j} = \partial F_i(\sigma)/\partial\sigma_j|_{\sigma=\sigma_0}$ linearized at the initial conductivity $\sigma_0$. The measurement noise $\mathbf{n}$ has noise covariance $\Sigma_n$.

To determine the $z$-score, we calculate the maximum likelihood expectation and standard deviation

$$\hat{m} = E[m] = A_R \Delta\hat{\sigma}_R \tag{5}$$

$$\text{std}^2(\hat{m}) = \text{var}(\hat{m}) = E[||m - \hat{m}||^2] = E[||\mathbf{R}_R\mathbf{n}||^2] = \mathbf{R}_R \Sigma_n \mathbf{R}_R{}^\intercal = (\mathbf{J}_R{}^\intercal \Sigma_n{}^{-1} \mathbf{J}_R)^{-1} \tag{6}$$

so that

$$z = \frac{\hat{m} - m_0}{\text{std}(\hat{m})} = \frac{A_R \Delta\hat{\sigma}_R}{\mathbf{R}_R{}^\intercal \Sigma_n{}^{-1} \mathbf{R}} = A_R \Delta\hat{\sigma}_R \sqrt{\mathbf{J}_R{}^\intercal \Sigma_n{}^{-1} \mathbf{J}_R} \tag{7}$$

The definition of Isaacson [4] and similar work is based on the norm of difference measurements $||\Delta\mathbf{d}||$. The formulation of (7), by contrast, takes into account the effects of the measurement scheme, noise properties of the system and multiple stimulation and measurement patterns. Using the definition of distinguishability as a cost function, an optimal stimulation and measurement scheme can be defined given a specific conductivity distribution and geometry.

## 3 Optimal Stimulation and Measurement

Generally, the optimal stimulation and measurement has been defined in terms of the L1-, L2-, or L-infinity norm of the measurement data. These interpretations do not account for the reconstruction scheme.

An alternative optimal stimulation and measurement criteria is to select stimulation and measurement patterns that satisfy the primal maximin or dual minimax criteria

$$\hat{\mathbf{x}} = \max_{\mathbf{x}} \min_{\mathbf{y}} \mathbf{y}^{\mathsf{T}} \mathbf{A} \mathbf{x} \qquad \text{(primal)} \tag{8}$$

$$\hat{\mathbf{y}} = \min_{\mathbf{y}} \max_{\mathbf{x}} \mathbf{y}^{\mathsf{T}} \mathbf{A} \mathbf{x} \qquad \text{(dual)} \tag{9}$$

where $\mathbf{A}$ is a matrix of distinguishabilities $z$ arranged such that each column represents a particular stimulation and measurement pattern and each row is a single ROI. The primal problem is to find the best stimulation and measurement strategy $\mathbf{x}$ that will maximize distinguishability $z$ given the ROIs $\mathbf{y}$ that provide minimum distinguishability. The dual of the problem can also be solved to find the ROIs that minimize the maximum distinguishability.

The primal optimization problem can be solved through linear programming by splitting the problem so that

$$\min_{\mathbf{y}} \mathbf{y}^{\mathsf{T}} \mathbf{A} \mathbf{x} = \min_{i} \mathbf{e}_i{}^{\mathsf{T}} \mathbf{A} \mathbf{x} \quad \rightarrow \quad v \leq \mathbf{e}_i{}^{\mathsf{T}} \mathbf{A} \mathbf{x} \tag{10}$$

and then maximizing the minimum value $v$

$$\begin{gathered} \max_{\mathbf{x}} \min_{\mathbf{y}} \mathbf{y}^{\mathsf{T}} \mathbf{A} \mathbf{x} \quad \rightarrow \quad \max v \\ \text{such that} \quad v\mathbf{e} - \mathbf{A}\mathbf{x} \leq 0; \quad \mathbf{e}^{\mathsf{T}}\mathbf{x} = 1; \quad \mathbf{x} \geq 0 \end{gathered} \tag{11}$$

or in matrix form

$$\begin{bmatrix} -\mathbf{A} & \mathbf{e} \\ \mathbf{e}^{\mathsf{T}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ v \end{bmatrix} \begin{matrix} \leq & 0 \\ = & 1 \end{matrix} \tag{12}$$

with $\mathbf{x} \geq 0$ and the optimal distinguishability $v$ is a free variable. $\mathbf{e}$ is the all ones column vector. $\mathbf{e}_i$ is the basis vector which is all zeros except for a single 1 at row $i$. The solution $\hat{\mathbf{x}}$ gives the ratio of stimulation and measurement strategies that will result in optimal distinguishability.

The dual problem can be solved similarly

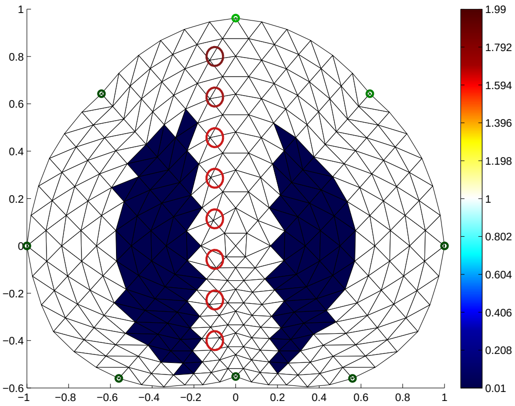
$$\begin{gathered} \min_{\mathbf{y}} \max_{\mathbf{x}} \mathbf{y}^{\mathsf{T}} \mathbf{A} \mathbf{x} \quad \rightarrow \quad \min u \\ \text{such that} \quad u\mathbf{e} - \mathbf{A}^{\mathsf{T}}\mathbf{y} \geq 0; \quad \mathbf{e}^{\mathsf{T}}\mathbf{y} = 1; \quad \mathbf{y} \geq 0 \end{gathered} \tag{13}$$

with $\mathbf{y} \geq 0$ and the optimal distinguishability $u$ is a free variable. The optimal distinguishability will be the same for both primal and dual problems ($v$ and $u$ respectively) if there is a saddle point in the solution space.

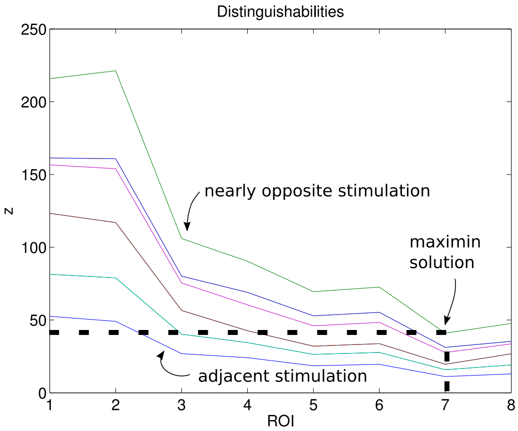
# 4 Results

Finite element simulations were performed in EIDORS version 3.5 [3] in combination with implementations of the previously described distinguishability (7) and linear programming (11), (13) equations.

In these simulations, particular strategies were compared from adjacent to nearly opposite stimulation electrodes. Opposite stimulations were excluded since they have half the number of unique data. Figure 1a shows the $H_0$ FEM model of the human thorax with the ROI shown as red circles. Figure 1b shows the distinguishability $z$ as a function of ROI. This plot clearly shows that nearly opposite stimulation and measurement results in significantly better distinguishability, and thus, the minimax solution always chooses 1) the central ROI as the worst ROI ($\mathbf{y}_7 = 1.0$) and 2) the nearly opposite stimulation and measurement pattern ($x_7 = 1.0$) as the best strategy where strategies were numbered by number of electrodes between stimulation electrodes.




(a) FEM mesh used for simulations; ROI as red circles with darkest circle as ROI#1; central ROIs have lower distinguishability




(b) Distinguishability for various stimulation and measurement strategies from adjacent (light purple: universally lowest $z$) to nearly opposite (light green: universally highest $z$)

# 5 Discussion

This work demonstrates the use of linear programming to select the optimal stimulation and measurement patterns for a given geometry and conductivity distribution based on the distinguishability of conductivity changes within specified ROIs.

This work can be extended to robust stimulation patterns by first selecting an optimal set of stimulation patterns, then removing an electrode and repeating the process. The distinguishability and linear programming algorithm was demonstrated on arbitrary domains with arbitrary initial conductivity distributions. The granularity of the stimulation and measurement patterns, from single tetra-polar electrode measurements to arbitrary stimulation and measurement strategies, is independent of the algorithm. More interesting results will come from these sorts of stimulation and measurements because the maximin solution will not consist of a single stimulation pattern.

In summary, this work provides a practical algorithm for selecting appropriate and robust stimulation and measurement patterns on arbitrary domains.

# References

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