ESTIMATING ELECTRODE MOVEMENT

in Two Dimensions

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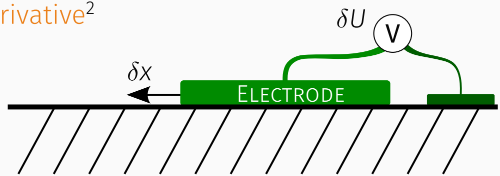
Jacobian Methods $J_{m,x} = \frac{\delta U}{\delta x}$

1. Naïve perturbation
2. perturbation
3. rank-one update[1]
4. Fréchet derivative[2]



[1] Gómez-Laberge C, Adler A. *Physiol Meas* **29**(6):S89–S99, 2008

[2] Dardé J, Hakula H, Hyvönen N, et al. *Inverse Problems and Imaging* **6**(3):399–421, 2012

- explore sensitivity of the direct perturbation method
- understand Fréchet derivative and build an implementation
- ask the Engineering question:

Do these methods give the same answers for a simplified model?

Movement In our case, two-dimensional tangential electrode movement on a fixed boundary.

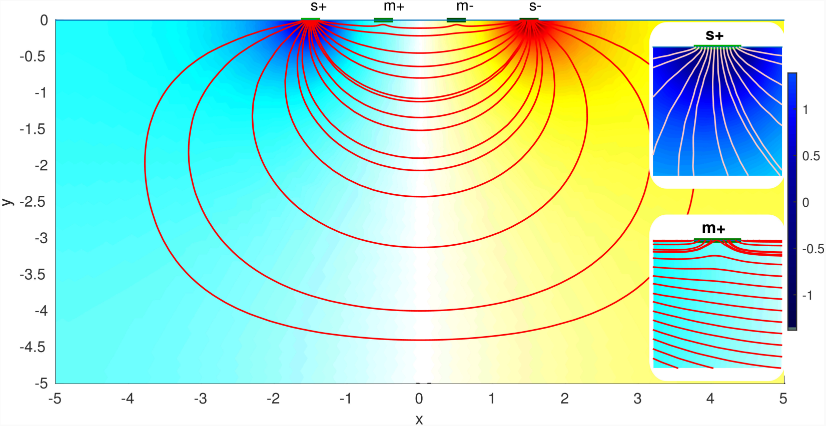
Jacobian An estimate of the local slope.

For Gauss-Newton updates, the Jacobian, in part, determines search direction δx, followed by a line search for distance α. If sufficiently linear, then $\alpha = 1$ can work well.

$$\delta x = (J^T J + \lambda^2 R^T R)^{-1} J^T U \tag{1}$$

$$x_{n+1} = x_n + \alpha \, \delta x \tag{2}$$

A TWO-DIMENSIONAL HALF-SPACE MODEL

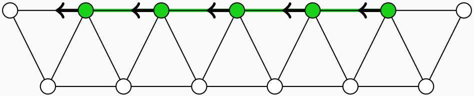


h_{max} = 0.1 m $\quad d_{e\ell}$ = 0.2 m $\quad z_C$ = 0.02 $\Omega \cdot$m $\quad \sigma = 1$ S/m

Naïve Perturbation

`Naïve Perturbation`

$$J_m = \frac{\delta U}{\delta x}$$

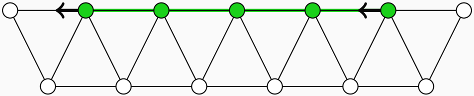


all electrode nodes are perturbed

Perturbation

`Perturbation`

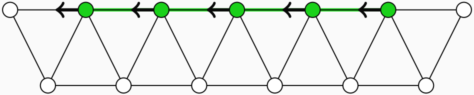
$$J_m = \frac{\delta U}{\delta x}$$



electrode *boundary* nodes are perturbed

Rank-One Update Gómez-Laberge

$$J_m = \frac{\delta U}{\delta x}$$



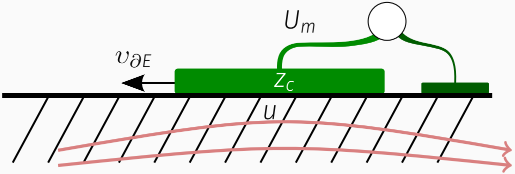
“the discrete form of the Lie derivative of the conductivity tensor”; the matrix rank-one update to the conductivity Jacobian due to FEM node movement

Crabb, Boyle

accounts for contact impedance z_c effects
in the Complete Electrode Model

$$\delta U = J_m h + O(h^2)$$

$$\mathrm{J}_m\ h = \frac{1}{z_c} \int_{\partial E} (h \cdot \upsilon_{\partial E})(U_m - u)(V_m - v) ds$$



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Crabb, Boyle

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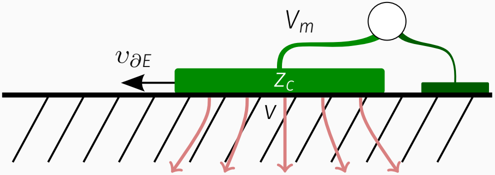

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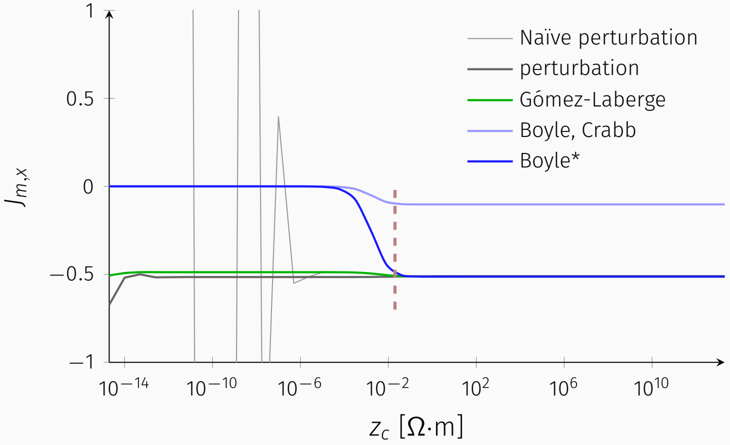
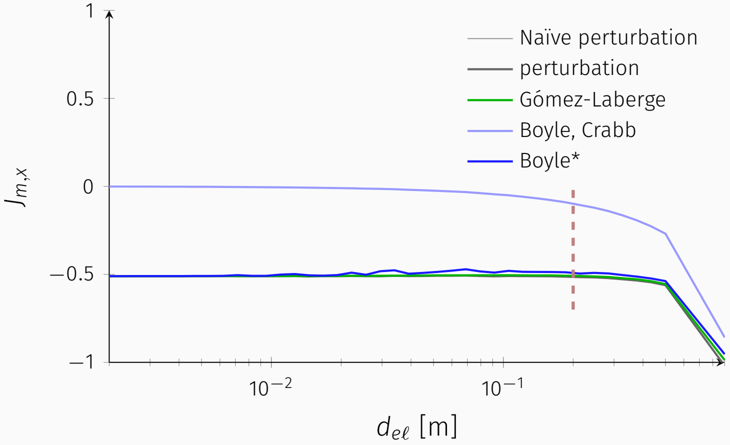
Boyle*

What if the contact impedance z_c units are wrong:
[Ω·m] rather than [Ω]?

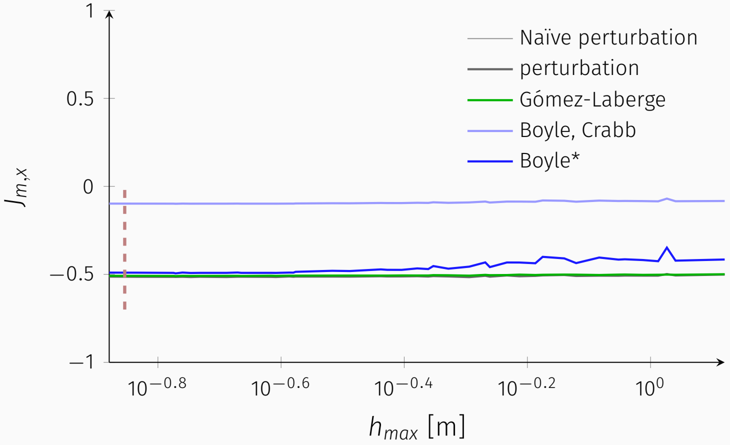


h_{max} = 0.1 m $d_{e\ell}$ = 0.2 m   plot for m+



h_{max} = 0.1 m z_C = 0.02 Ω·m plot for m+

MESH DENSITY h_{max} FOR THE M+ ELECTRODE



$d_{e\ell}$ = 0.2 m z_C = 0.02 Ω·m plot for m+

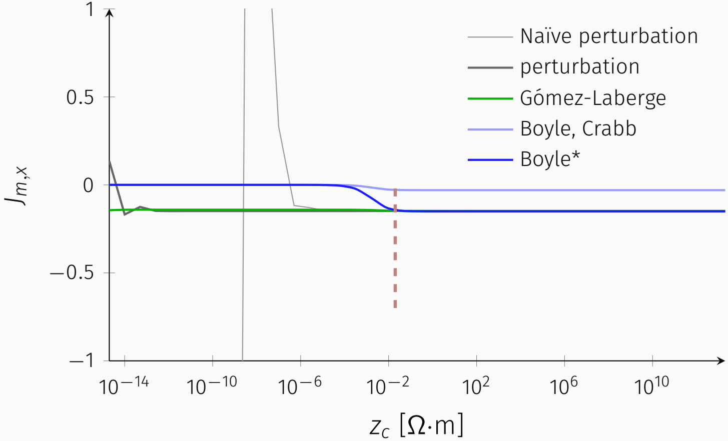
DISCUSSION

What does this mean for movement solutions?

- Fréchet deriv. is fast
  (forward solutions reused from *adjoint method*/conductivity)
- Fréchet deriv. magnitudes differ from perturbation methods
  (sign is correct/stable;
  hyper-parameter λ and/or line search α values will be different)
  (contact impedance units?)
- Fréchet deriv. results collapse for small contact impedance
  $z_c \ll \Omega_\sigma$, and more sensitive to mesh density h_{max} than
  conductivity solutions
  (use "rank-one update" method when necessary?)

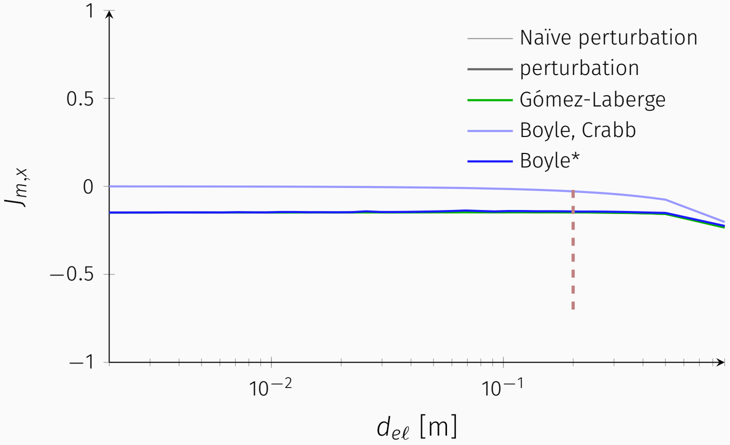
Questions?

CONTACT IMPEDANCE Z_C FOR THE S- ELECTRODE



h_{max} = 0.1 m $d_{e\ell}$ = 0.2 m   plot for s-

ELECTRODE DIAMETER $d_{e\ell}$ FOR THE S- ELECTRODE



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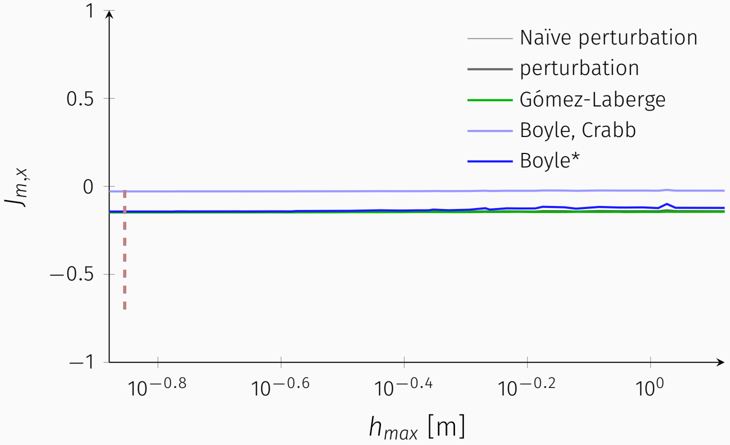

$d_{e\ell}$ = 0.2 m z_C = 0.02 Ω·m   plot for s-