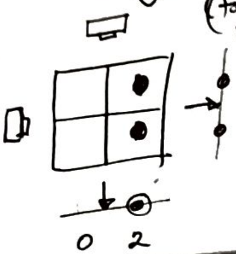
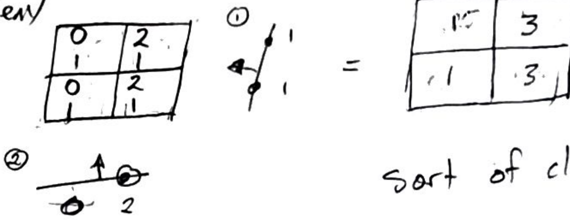
Pictures and CT and, eventually EIT
Electrical Impedance Tomography .

Projections (forward problem) | **Back Projection** (inverse problem)

ex1

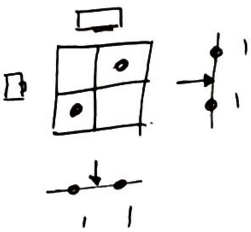
photographs



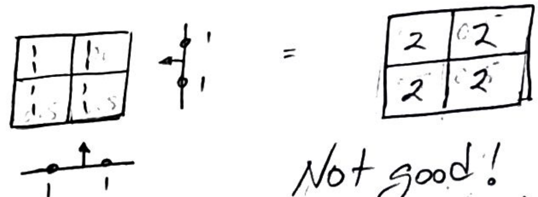


sort of close.

ex2



uhoh.



Not good!
What happened?

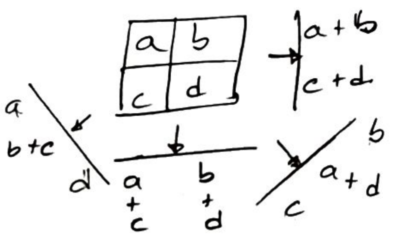
ambiguous. measurements

two equally valid solutions

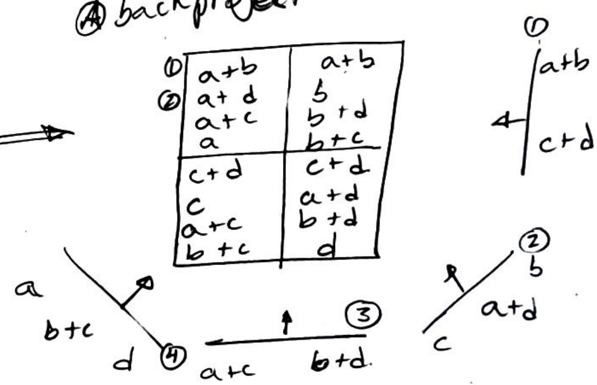
give same measurements.!

~~E~~ How can we solve the ambiguity ?

More angles.

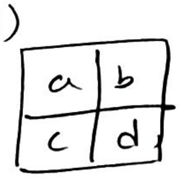


Ⓐ backproject

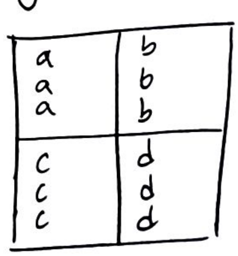


Ⓒ divide by projections minus one.

$\frac{x}{(4-1)}$



Ⓑ subtract projection sum : $-(a+b+c+d)$



!

Apr 4, 2018 ②

Will MANY more angles make a perfect picture?

No.: in the limit (continuous data), we still have measurement noise and imperfect reconstruction algorithms.

Error Sources
- discrete data
- measurement noise (statistical errors).
- systematic errors (detector misalignment)
- ✱ - incomplete data
- ✱ - scattering, absorption

Recall For CT, one approach is filtered back projection (FBP)

FBP

$$f(x,y) = \int_0^{\pi} d\phi \left[\int_{-\infty}^{\infty} d\omega \, |\omega| P(\omega) \, e^{2\pi j \omega s} \right]_{s = x\cos\phi + y\sin\phi}$$

1) FT of projection $p(s,\phi) \xrightarrow{FT} P(\omega)\, e^{j2\pi\omega s}$
2) Multiply by frequency filter : ramp $|\omega|$

- minimize blurring (low freq.)
- maximize contrast (high freq).

3) Inverse FT
4) Back project
5) Sum over all filtered back projections.

$\rightarrow$ complete data, no noise + FBP = original 2D image.

Another way: Maximum Liklihood, Expectation Maximization.
or
Gauss - Newton (Iterative)

fwd. $m = Jx$
$J^T m = (J^T J)x$
$(J^T J)^{-1} J^T m = (J^T J)^{-1} (J^T J) x$
$(J^T J)^{-1} J^T m = x$ inv

$$\underbrace{x_{n+1}}_{\text{next estimate}} = \underbrace{x_n}_{\text{last estimate}} - \underbrace{(J^T J)^{-1} J^T}_{\text{pseudo-inverse } (J^+)} \underbrace{b_n}_{\text{error}} \; ; \quad b_n = \underbrace{F(x_n)}_{\text{forward model}} - \underbrace{m}_{\text{measurements}} \; ; \quad \underbrace{x_0}_{\text{initial guess}}$$

Jacobian

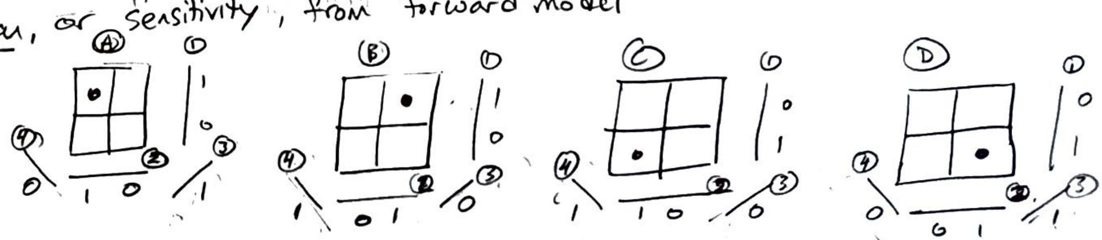
simulate measurements.

... for PET, SPECT, CT, EIT

Jacobian, or "sensitivity", from forward model

a "perturbation"

$$J_{ij} = \frac{\partial m_i}{\partial x_j}$$



$$J = \begin{array}{c} \\ \\ \end{array}
\begin{array}{cccc} Ⓐ & Ⓑ & Ⓒ & Ⓓ \end{array}$$

$$J = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{array}{l} \big\} ① \\ \\ \big\} ② \\ \\ ③ \\ ④ \end{array}$$

ex $m_A = J x_A = F(x_A)$

for $x_A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

✱ Remember linear algebra?

Try it!

Initial guess: $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \phi$; Measurements A ; solution A
"empty"

$x_1 = x_0 - (J^TJ)^{-1} J^T b_n$

$x_1 = x_0 - (J^TJ)^{-1} \left[J^T (J x_0 - m_A) \right]$; $m_A = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = J x_A$

$x_1 = \quad + (J^TJ)^{-1} J^T m_A$

$x_1 = \underbrace{(J^TJ)^{-1} J^T J}_{I} x_A$

$x_1 = x_A$ $\longrightarrow$ next iteration error $= \phi$, stop!

BUT noise + errors means $m_A \neq J x_A$,
iterations minimize the error

Back projection is $x = J^T m_A$

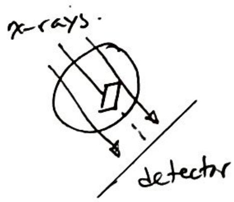
Filter is $J^TJ = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix}$ spatial low pass filter

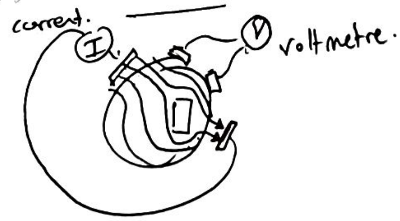
$(J^TJ)^{-1} \simeq \frac{1}{10} \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}$ spatial high pass filter

★ this is type #2 filtered back projection.

CT versus EIT

★ We made it, on to the pictures!





★ density

★ conductivity.

- individual x-rays go in a straight line.

- current flows everywhere
- small changes in measurements can indicate large changes in conductivity

— measurement noise —

for EIT $x_{n+1} = x_n + (J^T W J + \lambda^2 R)^{-1} (J^T W b_n + \lambda^2 R (x_* - x_n))$

inverse measurement covariance — regularization. — prior estimate.

Electrical Impedance Tomography

Alistair Boyle

School of Electrical Engineering and Computer Science
University of Ottawa

Systems and Comptuer Engineering
Carleton University

Apr 4, 2018

Electrical Impedance Tomography

swisstom

Typical EIT Bedside Equipment; Ventilator Management
Swisstom BB2

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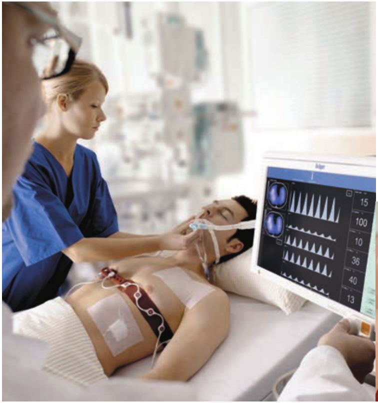
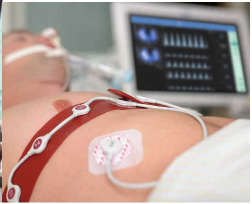
images from Swisstom BB2 brochure, retrieved Apr 3, 2018

Electrical Impedance Tomography





Typical EIT Bedside Equipment; Ventilator Management
Dräger Pulmovista 500

images from Dräger Pulmovista 500 brochure, retrieved Apr 3, 2018

Electrical Resistivity Tomography

Typical ERT Survey Equipment
ABEM Terrameter LS

Guideline Geo, technical specs retrieved Feb 22, 2018

PBG Geophysical Exploration Ltd., image retrieved Feb 22, 2018

Electrical Resistivity Tomography

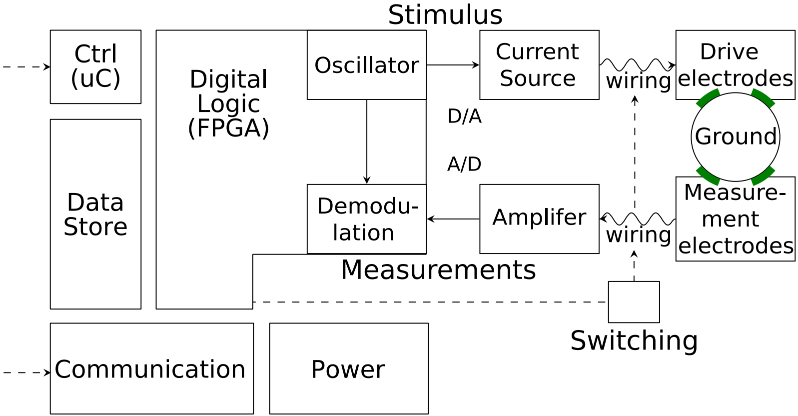
Long-term remote monitoring

slow moving landslide at Hollin Hill, UK with colleagues from the British Geological Survey

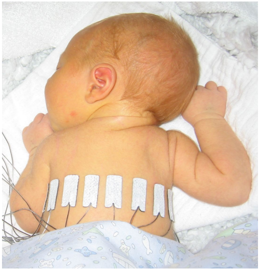
daily measurements 2008–present

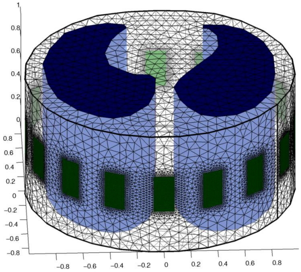
Automated Landslide Electrical Resistivity Tomography (ALERT) system

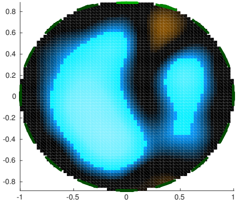
Electrical Resistivity Tomography



Electrical Impedance Tomography





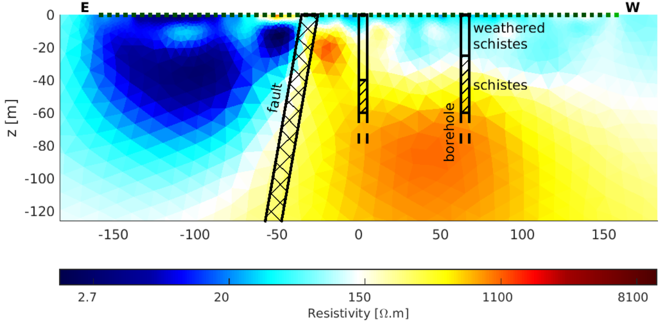


Typical EIT Reconstruction

Neonate Lungs

EIDORS tutorial: GREIT Reconstruction for an neonate human thorax geometry

Electrical Resistivity Tomography



Typical ERT Survey
Pont-Péan, France

A. Boyle, *Geophysical Applications of Electrical Impedance Tomography*, PhD thesis, 2016

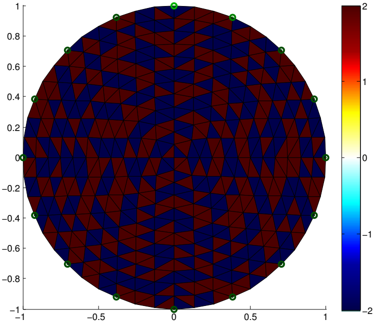
Methods

Absolute imaging problem; large conductivity contrasts
... a Gauss-Newton nonlinear iterative solver

$$\min_x ||\mathbf{Ax} - \mathbf{b}||_2^2 \quad (1)$$

$$\delta \mathbf{x}_n = -(\mathbf{J}_n^\mathsf{T} \mathbf{J}_n)^{-1} (\mathbf{J}_n^\mathsf{T} \mathbf{b}) \quad (2)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \alpha_{n+1}\, \delta \mathbf{x}_{n+1} \quad (3)$$



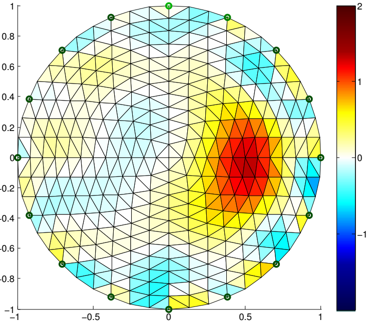
Methods

Absolute imaging problem; large conductivity contrasts
... a Gauss-Newton nonlinear iterative solver

$$\min_x ||\mathbf{A}\mathbf{x} - \mathbf{b}||^2_{\mathbf{W}} + ||\lambda \mathbf{R}(\mathbf{x} - \mathbf{x}_*)||^2_2 \tag{1}$$

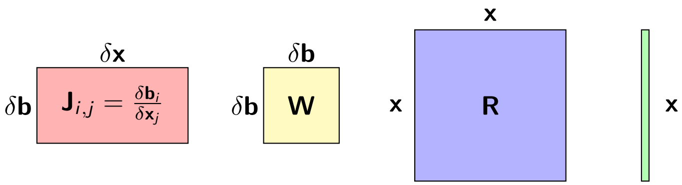
$$\delta \mathbf{x}_{n+1} = -(\mathbf{J}_n^\mathsf{T}\mathbf{W}\mathbf{J}_n + \lambda^2 \mathbf{R}^\mathsf{T}\mathbf{R})^{-1}(\mathbf{J}_n^\mathsf{T}\mathbf{W}\mathbf{b} - \lambda^2 \mathbf{R}^\mathsf{T}\mathbf{R}(\mathbf{x}_n - \mathbf{x}_*)) \tag{2}$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \alpha_{n+1}\, \delta \mathbf{x}_{n+1} \tag{3}$$

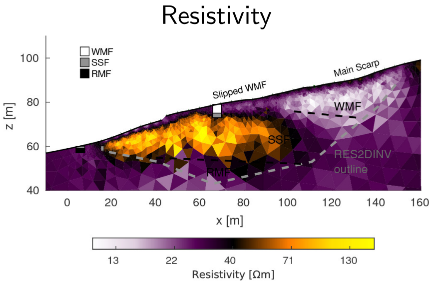


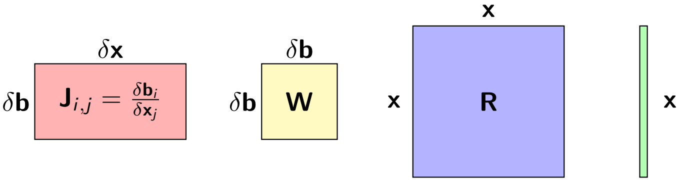
Methods

$$\delta \mathbf{x}_{n+1} = -(\mathbf{J}_n^\intercal \mathbf{W} \mathbf{J}_n + \lambda^2 \mathbf{R}^\intercal \mathbf{R})^{-1} (\mathbf{J}_n^\intercal \mathbf{W} \mathbf{b} - \lambda^2 \mathbf{R}^\intercal \mathbf{R}(\mathbf{x}_n - \mathbf{x}_*))$$



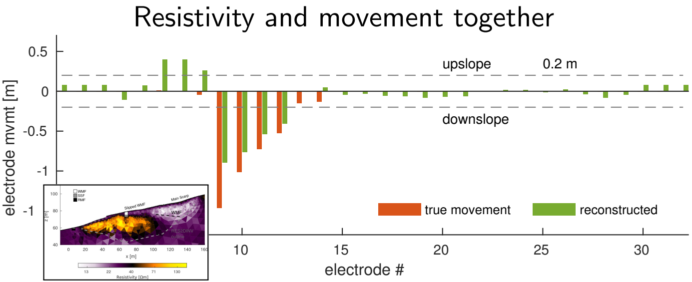
Methods

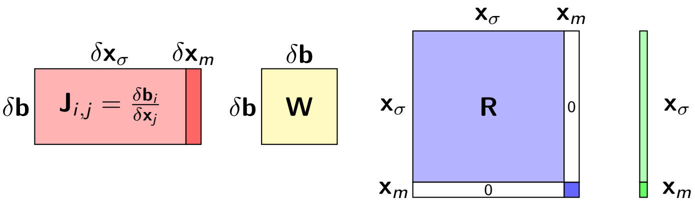




A. Boyle, P. Wilkinson J. Chambers, P. Meldrum, S. Uhlemann A Adler, Jointly reconstructing ground motion and resistivity for ERT-based slope stability monitoring, *Geophysical Journal International*, 212(2), 2018

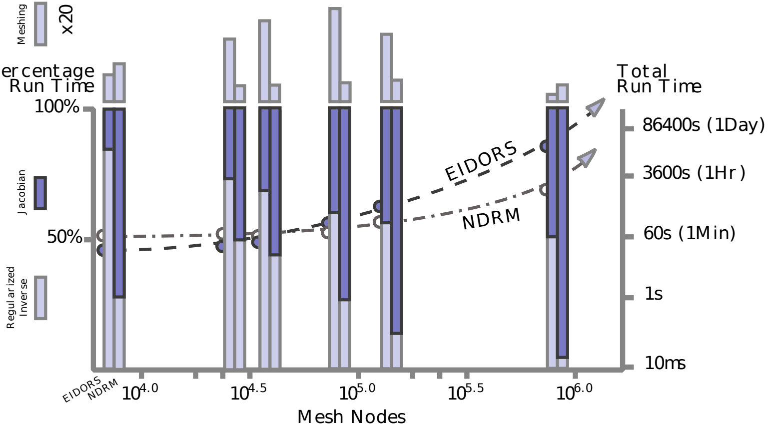
Methods





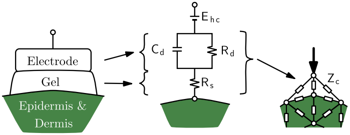
A Boyle, P Wilkinson J Chambers, P Meldrum, S Uhlemann, A Adler, Jointly reconstructing ground motion and resistivity for ERT-based slope stability monitoring, *Geophysical Journal International*, 212(2), 2018

3D

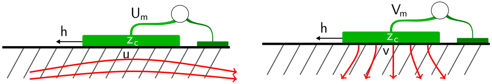


A Boyle, A Borsic, A Adler, Addressing the Computational Cost of Large EIT Solutions, *Physiological Measurement*, 33(5), 2012

2 weeks



2 days



(a) Measurement (b) Stimulus

2 hours

runtimes

A Boyle, A Adler, Impact of Electrode Area, Contact Impedance and Boundary Shape on EIT Images, *Phys. Meas.*, 32(7), 2011

A Boyle, M Crabb, M Jehl, W Lionheart, A Adler, Methods for Calculating the Electrode Position Jacobian for Impedance Imaging, *Phys. Meas.*, 38(3), 2017

Average 565 derailments per year, 80 with dangerous goods (Canada, 2010-2015)[2]
Gogama clean-up costs will be "in the millions" – MPP F. Gelinas[3]

[1] Transportation Safety Board of Canada, *Railway Investigation Report R13E0069*, Apr 2013

[2] Transportation Safety Board of Canada, *Statistical Summary - Railway Occurrences 2015*, Feb 2016

[3] M. Stackelberg, CBC News, *Ontario bills CN $350K for Gogama derailment clean-up*, Dec 2015

Mount Polley Mine, Likely, BC: spilled 4,500,000 m^3 of tailings[4] with clean up costs of $200–500 mil.[5] (2014)
46 “dangerous or unusual occurances” 2000–2012 in BC[6]; 2–5 “major” tailings dam failures per year[7]

[4] Indep. Expert Eng. Invest. & Review Panel, *Report on Mount Polley Tailings Storage Facility Breach*, 2015

[5] CBC News, *Mount Polley mine tailings spill*, Aug 2014

[6] G. Hoekstra, Vancouver Sun, *Liberals keeping dangerous occurrences at B.C. tailings ponds a secret*, Aug 2014

[7] M. Davies, et al., *Mine Tailings Dams: When Things Go Wrong*, AGRA Earth & Env. Ltd, 2002

What is the Systems Problem?

Long-term remote monitoring is a

- hard systems problem, and
- vital for Canada

Long-term, reliable remote monitoring can mitigate risks and enable timely response

flickr: druclimb, Toe of the Katzie Glacier, near Vancouver, BC, 2008

Could manage *ground stability* risks with

- a tool for real-time monitoring (prediction) of movement
- robust, reliable, informative reconstructions

Tool of choice:

Electrical Resistivity Tomography
Electrical Impedance Tomography

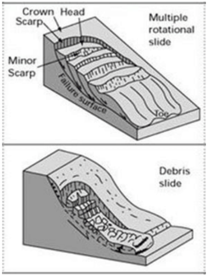


image: A Pitasi, Phd Thesis, Mediterranean University of Reggio Calabria, 2016

Could manage *ground stability* risks with

- a tool for real-time monitoring (prediction) of movement

  electrode movement & resistivity

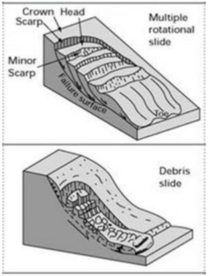
- robust, reliable, informative reconstructions

  instrument, data, algorithm, implementation

Tool of choice:

Electrical Resistivity Tomography

Electrical Impedance Tomography



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