Propagation of Measurement Noise into Images

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Abstract: We examine the relationship between measurement noise and reconstructed images. For linearised EIT reconstruction, the location and distribution of artefacts can be identified exactly for known noise distributions, or approximately for arbitrary distributions. These intricate artefact models help explain how experienced users can often identify "bad" measurements in real-world data.

1 Introduction

Noise exists in all real-world data: undesired fluctuations in measured signal. There are a wide variety of noise sources, many of which are not well modelled by Gaussian distributions. For example certain models of quantization, clipping, burst, thermal, and shot noise are non-Gaussian. EIT measurement noise affects reconstructed conductivity images leading to artefacts. Structured noise can result in persistent artefacts which are difficult to identify from single images and are not removed by temporal filtering.

2 Methods

A single-step Gauss Newton EIT reconstruction matrix $\mathbf{Q}$ for measurements $\mathbf{b}$ contaminated by additive (non-parametric) noise η gives an image $\mathbf{x}$ where

$$\begin{aligned}\mathbf{x} &= (\mathbf{J}^\mathsf{T}\mathbf{W}\mathbf{J} + \lambda^2\mathbf{R})^{-1}\mathbf{J}^\mathsf{T}\mathbf{W}(\mathbf{b}+\eta) = \mathbf{Q}(\mathbf{b}+\eta) \qquad (1)\\ &= \mathbf{x}^{(\mathbf{b})} + \mathbf{x}^{(\eta)} \quad \text{with } \mathbf{x}^{(\mathbf{b})} = \mathbf{Q}\mathbf{b}\,; \quad \mathbf{x}^{(\eta)} = \mathbf{Q}\eta\end{aligned}$$

for Jacobian $\mathbf{J}$, inverse measurement covariance $\mathbf{W}$, and regularization $\mathbf{R}$ adjusted by hyperparameter λ. The uncontaminated measurements $\mathbf{b}$ give the image $\mathbf{x}^{(\mathbf{b})}$, while noise generates a distribution of additive changes to the image $\mathbf{x}^{(\eta)}$.

To calculate the distribution of noise afflicted images, a typical technique is the Bootstrap Method [1], where the mean of all images $\hat{\mathbf{x}}^{(\eta)}$, after random sampling of measurement noise η with replacement, gives a generalized distribution for sampled noise η_j where $\hat{\mathbf{x}}^{(\eta)} = \frac{1}{m}\sum_j^m \mathbf{Q}_{/j}\ \eta_j$ over many noise samples $m \gg 100$ will tend to a Gaussian distribution courtesy of the central limit theorem. With many measurements, Leave-One-Out (LOO) will give similar results to the bootstrap.

We see immediately that we need only calculate $\mathbf{x}^{(\mathbf{b})}$ once and in fact, it only shifts our distribution's centre. We can see the individual components of our measurement noise translated into an image distribution as a linear combination of measurements. Furthermore, the bootstrap resampling of measurements is not adding additional information, but is instead converting a point estimate into an approximately continuous distribution. If instead we examine each measurement's noise distribution individually and transform that distribution into the image domain through the reconstruction matrix $\mathbf{Q}$, we have an exact probability distribution for the reconstructed image $\mathbf{x}^{(\eta)} = \mathbf{Q}\eta = \mathbf{Q}\sum_k \mathbf{e}_k D_k = \sum_k \mathbf{x}_k^{(\eta)}$, for $\mathbf{e}_k = [0 \ldots 1 \ldots 0]^\mathsf{T}$ and 1 at the k row. The sum of the images gives a probabilistic "mixture model" which fully describes the variation in the reconstruction due to measurement noise.

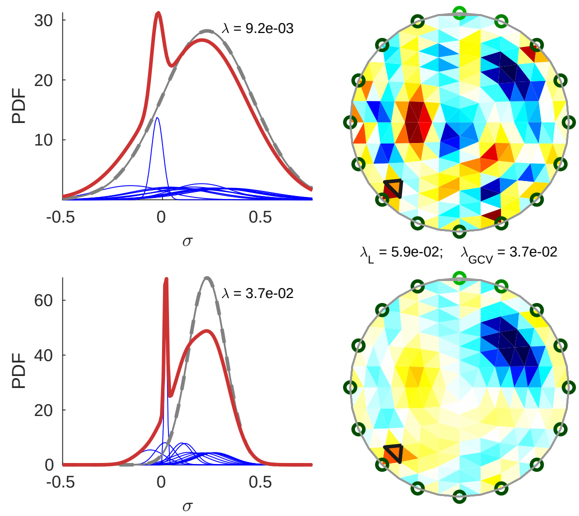

Figure 1: The PDF (blue) for a voxel (black triangle) changes depending on which measurements are dropped, in this case based on leaving out an electrode. Taking the sum of the PDFs (red) shows a distinct peak near the true conductivity. Removing single measurements (LOO) or bootstrap gives distributions driven by the central limit theorem (grey dashed, scaled to red line). Tikhonov regularized, GN-1 step images were reconstructed from noisy data $\eta = \mathcal{N}(0, c/10)$ and electrode#10 $\eta = \mathcal{N}(0, c)$ with $c = \text{std}(\mathbf{b})$. The linear mixture of distributions $\mathbf{Q}$ changes depending on regularization λ (upper versus lower images/plots). Reconstruction at λ for particular samples of noise *(right)* give a conductivity which is most often the max of the grey-dashed plot to the left, but the correct solution is at the red peak; *(above)* reduced regularization $\lambda_{GCV}/4$ produces distributions with greater variance, than *(below)* for λ_{GCV}; λ by L-curve λ_L and GCV λ_{GCV} are similar.

3 Observations and Conclusions

The effects of each measurement noise are separable and are captured by a scalar distribution. The scalar distributions form a linear mixture [2] through the reconstruction $\mathbf{Q}$ (Figure 1). The distribution of noise artefacts is solely a function of the reconstruction matrix $\mathbf{Q}$ and the noise distribution η. The combination of all k noise distributions D_k gives a specific image artefact distribution per voxel.

Greater regularization λ reduces noise variance in the overall mixture model, but does not necessarily "suppress" particular measurements. Noisy images are a mixture of scaled and offset distributions. Regions that are particularly sensitive to a single group of measurements allow isolation of that measurement's noise distribution and allow direct diagnosis of "bad" measurements.

This work demonstrates an efficient and direct method for computing the effect of noise on images and is an enabling tool for identifying noise distributions in artefact affected images using time-series data which may not be amenable to temporal filtering.

References

[1] Efron B. *The Annals of Statistics* **7**(1):1–26, 1979

[2] Richardson S, Green P. *J R Stat Soc Series B Stat Methodol* **59**(4):731–792, 1997