Transient Circuit Simulation of (Cole-Cole) Fractional-Order Models for Biomedical Instrumentation

Alistair Boyle
Carleton University, Ottawa, Canada
EIT2019, July 1-3, 2019

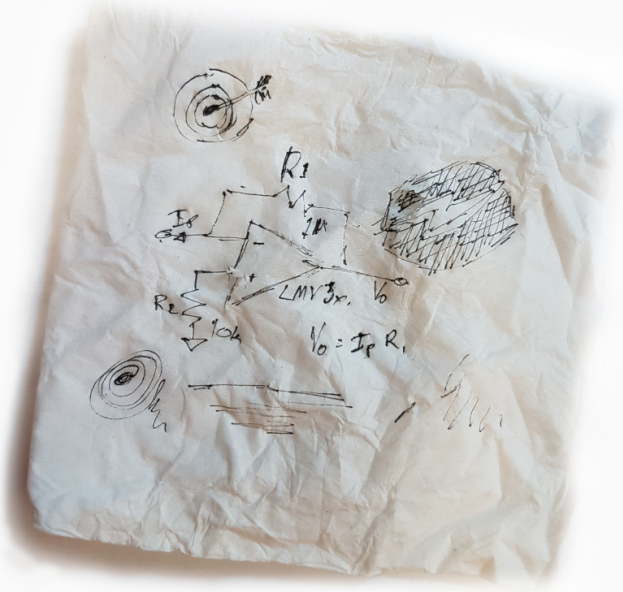
Question

Why do current sources not behave as *expected* when applied to some *biological* subjects?

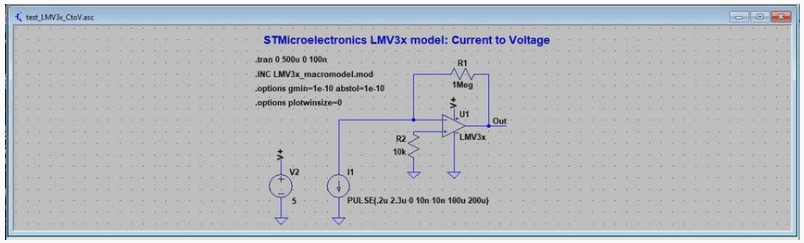
EXPECTATION

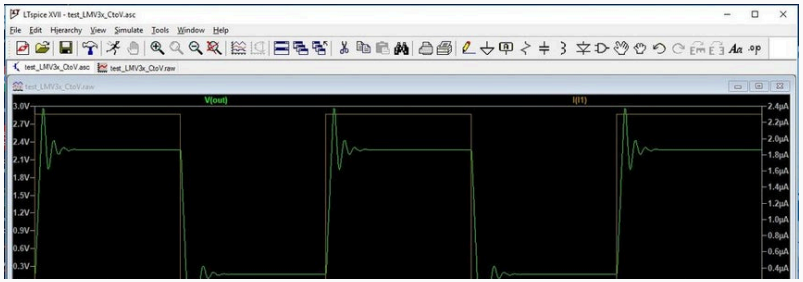
A) Circuit Design Tools

- A Design Process:
  1. Paper napkin
  2. Schematic Capture
  3. *SPICE circuit simulation*
  4. Layout
  5. Assembly and Test

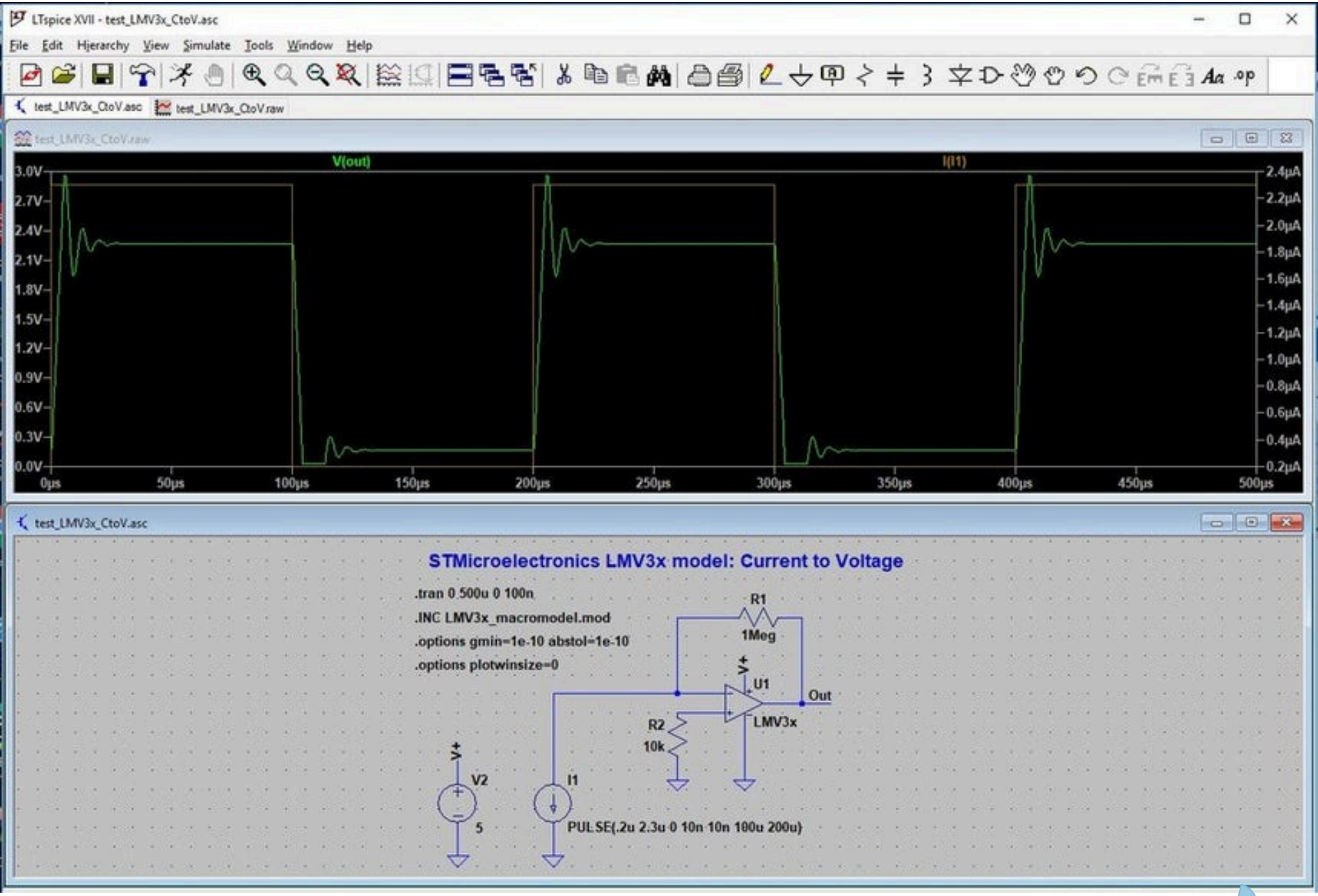


- Vendor model quality varies
- Not everyone simulates…



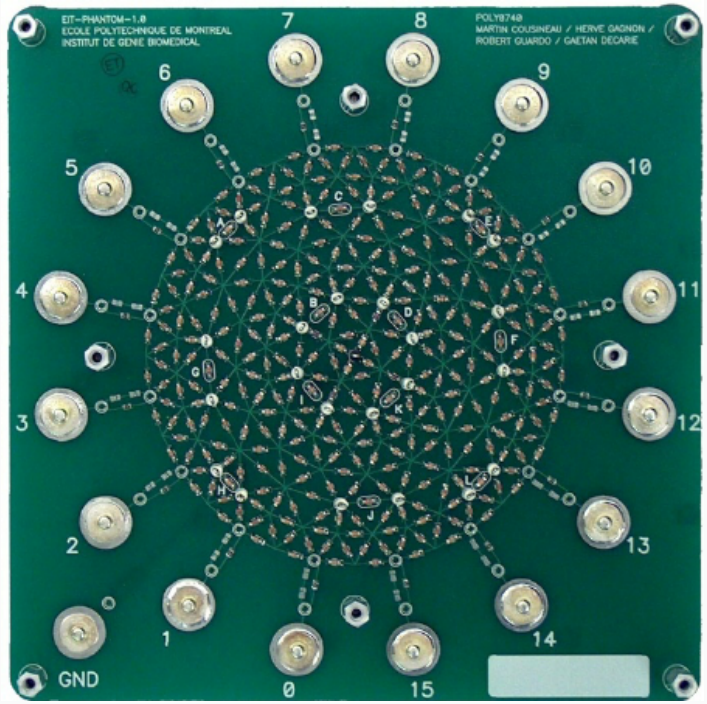


screenshot © instructables.com/RonFred

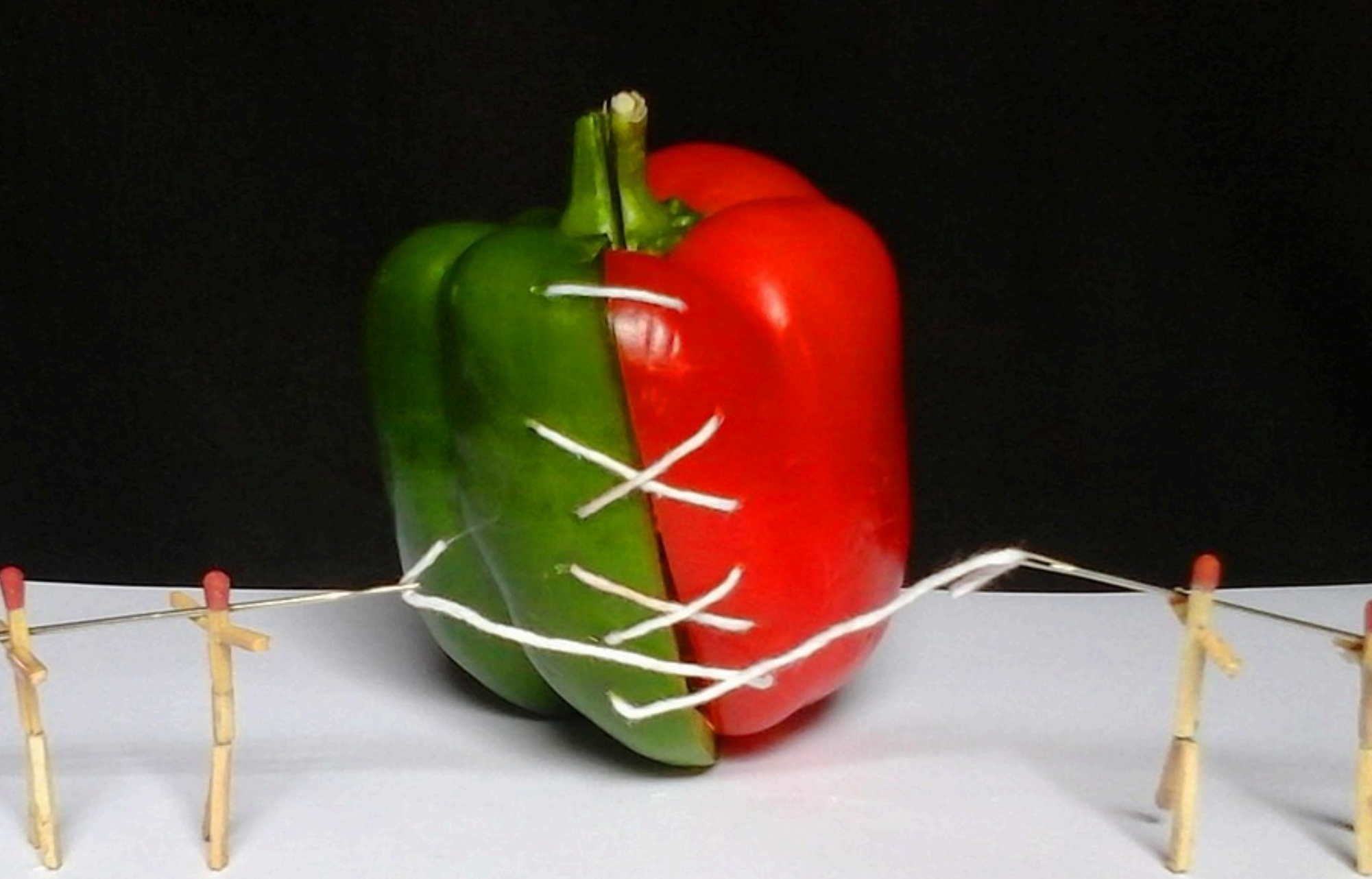


B) Test Subjects

- Linear phantoms: RLC networks ✓
- Non-linear phantoms: Active networks ✓
- Biological media: Cole-Cole type dispersions ?



H. Gagnon, et al., *A Resistive Phantom for the Assessing the Performance of EIT Systems,* IEEE Trans Biomed Eng (57)9, 2010.



FRACTIONAL MODELS

Cole-Cole Models

Biological media can be characterized as having *multiple dispersions*

$$\hat{\epsilon} = \epsilon_\infty + \sum_m \frac{\Delta\epsilon_m}{1 + (\mathbf{j}\omega\tau_m)^{(1-\alpha_m)}} + \frac{\sigma}{\mathbf{j}\omega\epsilon_0}$$

For complex permittivity $\hat{\epsilon}$,
with the permittivity of free space $\epsilon_0 = 8.8542 \times 10^{-12}$ F/m, angular frequency $\omega = 2\pi f$, and $\mathbf{j} = \sqrt{-1}$, the high frequency permittivity ϵ_∞, low-frequency ionic conductivity σ, time constants τ_m, breadth $0 \leq \alpha_m \leq 1$, and the permittivity difference $\Delta\epsilon_m = \epsilon_m - \epsilon_\infty$ for a particular permittivity ϵ_m at $\omega\tau_m \ll 1$.

Cole-Cole Models

Translated to conductivity

$$\sigma^* = \mathbf{j}\omega\epsilon_0\hat{\epsilon} = \sigma + \mathbf{j}\omega\epsilon_0\epsilon \quad \text{for} \quad \hat{\epsilon} = \epsilon + \frac{\sigma}{\mathbf{j}\omega\epsilon_0}$$

$$\sigma^* = \sigma + \mathbf{j}\omega\epsilon_0\left(\epsilon_\infty + \sum_m \frac{\Delta\epsilon_m}{1 + (\mathbf{j}\omega\tau_m)^{(1-\alpha_m)}}\right)$$

The term $(1 - \alpha_m)$ for $0 < \alpha_m < 1$ leads to a "fractional-order model."

Constant Phase Element (CPE)

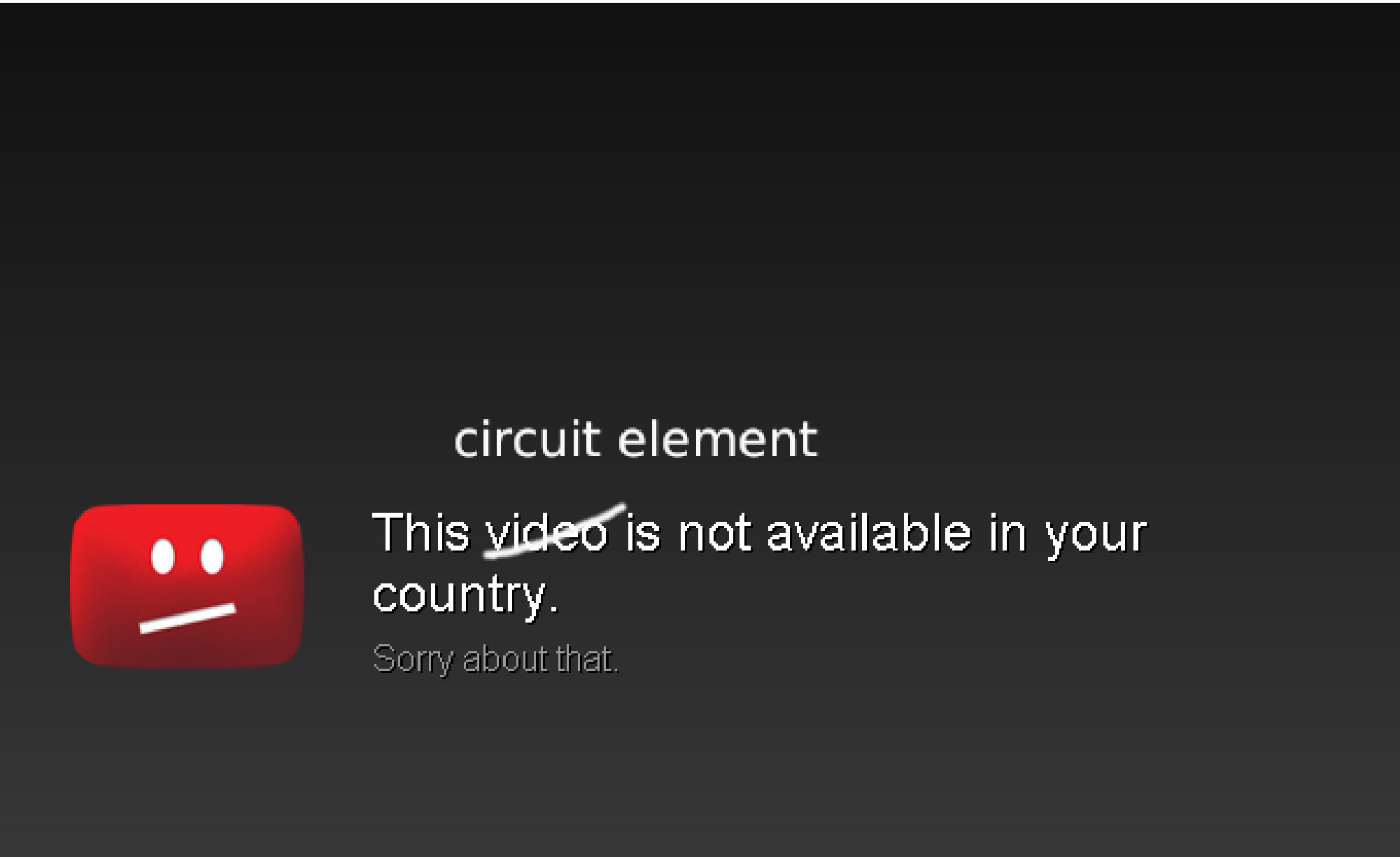
We can build a Cole-Cole circuit using a CPE

$$Y_{\text{CPE}} = Q_0(\mathbf{j}\omega)^{\beta} \qquad \text{for } Q_0 = \frac{1}{|Z|}$$

$$\text{at } \omega = 1 \text{ rad/s} \qquad \text{and with } 0 \leq \beta \leq 1$$

Constant phase is $-90\beta°$

$\beta = 1$ is an ideal capacitor, $\beta = 0$ is an ideal resistor



General CPEs are not standard SPICE elements.
Fractional-order models are often simulated in MATLAB.

Simulation

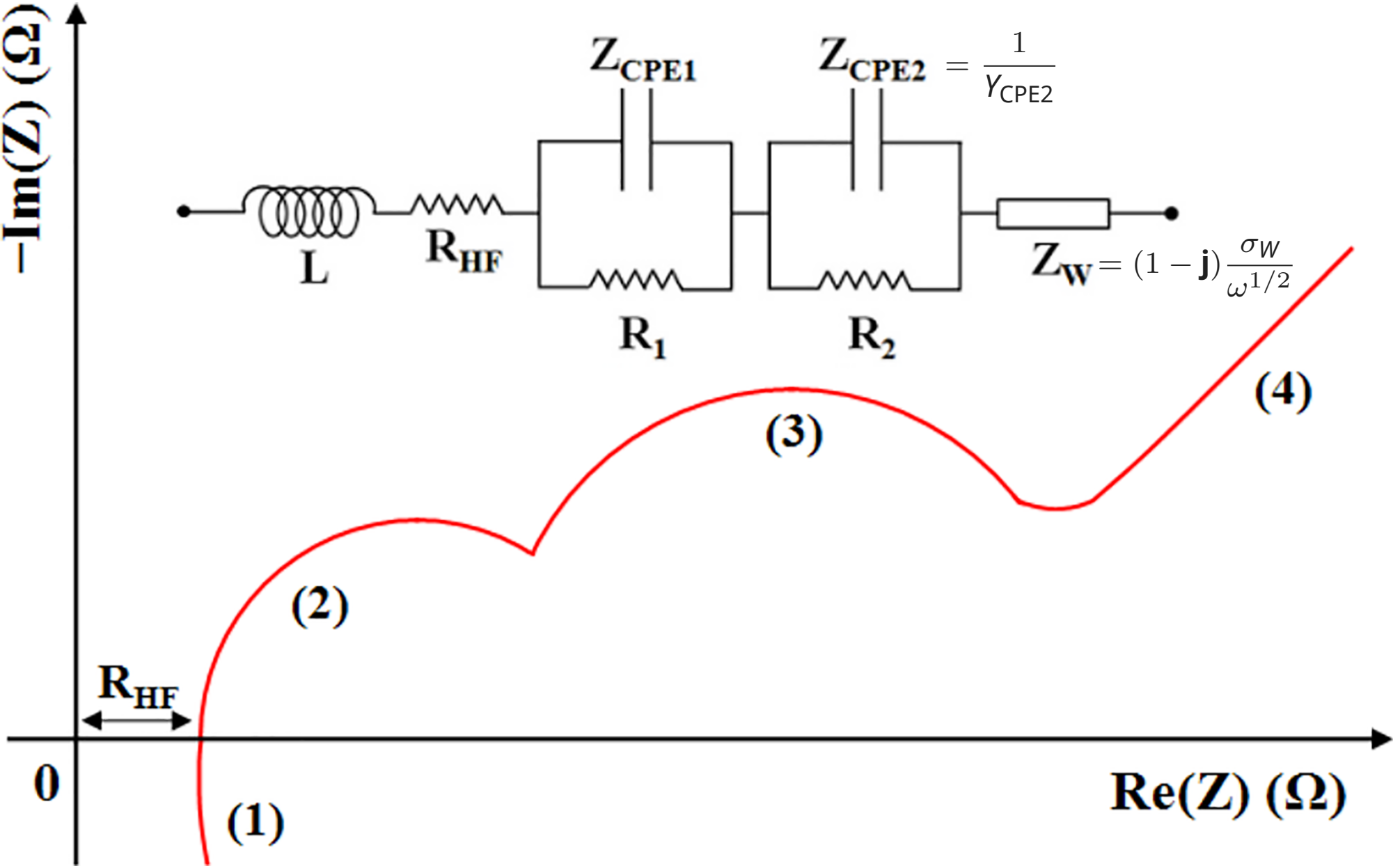
- MATLAB is not a common part of the design flow
- Vendor models are not available in MATLAB
- Approximate CPEs use many SPICE R-C elements
  (slows simulation significantly)
- Hard to scale up to FEM connectivity
  (many FEM elements, 3 or 6 edges per element,
  each edge contains a CPE…)
- No easy multifrequency model reduction when the FEM is inhomogeneous

Fractional-order Applications

Modelling of

- biological media,
- super capacitors,
- litium-ion batteries,
- photo-voltaic power performance (off-grid, mobile, low-power),
- control systems,
- chaotic systems,
- fractional-order filters, and
- ultra-low voltage sensors.

All suffer from the lack of circuit design tools and are being actively researched; CPE is the key element.



Nyquist plot and equivalent circuit of a lithium-ion battery cell

M. Grossi, B. Riccó, *EIS for biological analysis and food characterization: A review*, J. Sens. Sens. Syst. (6), 2017.

STRATEGY

Strategies for Handling CPEs

- Model reduction †
  (single frequency).
- Direct time-domain simulation by
  - time-varying convolution, or
  - state-space modelling.
- Direct frequency-domain simulation by
  - iterative numerical refinement, or
  - approximate RC-ladders
    (limited frequency range).

† A. Boyle, A. Adler, *Integrating Circuit Simulation with EIT FEM Models*, EIT2018, 2018.

EIDORS function `eit_spice` provides model reduction at a single frequency into a SPICE RC `subckt` netlist.

A Real Problem?

Question 1:
How and why do SPICE simulations not match on biological media?

A hypothesis (Cole-Cole) models.

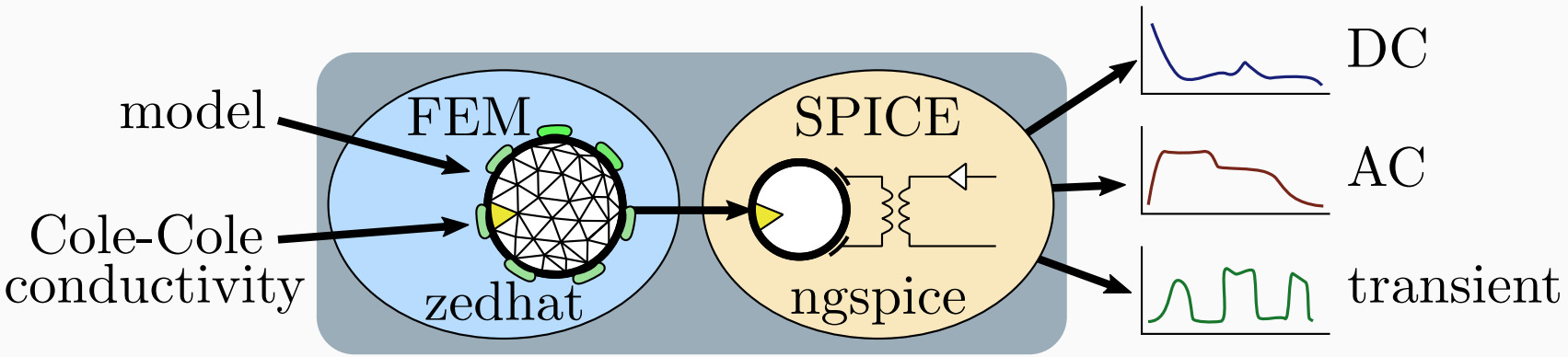
A hardware test platform† for current sources.

Question 2:
How would we address this annecdotally reported issue?

...

† A. Boyle, *Do current sources perform to "spec," as simulated? Hardware for evaluating simulator accuracy*, EIT2019, 2019.

The Path Forward

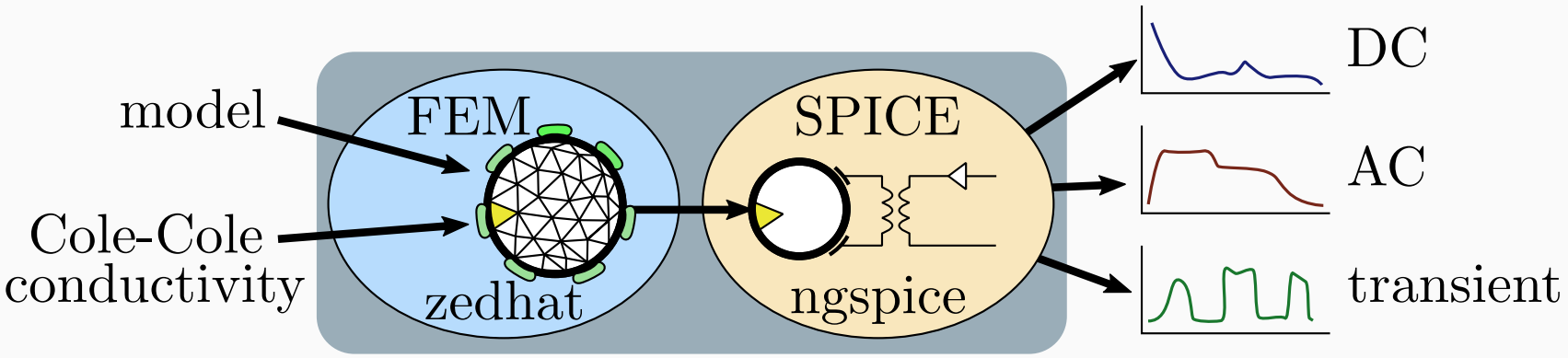


zedhat is a new open source EIT simulator †

ngspice is an open source SPICE simulator

† A. Boyle, *zedhat: an EIT tool library*, EIT2019, 2019.

The Path Forward



leverage *zedhat* to link EIT mesh, CEM and interior Cole-Cole models of biological materials to SPICE sim

leverage well-understood *ngspice* analyses (DC, AC, tran, etc) and incorporate complete electronics models