Transient Circuit Simulation of (Cole-Cole) Fractional-Order Models for Biomedical Instrumentation

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Abstract: In an ongoing process of interviewing designers of EIT and ERT instruments, realizing AC current sources that perform well *in situ* has been identified as a major source of instrument design challenges due to broadband switching transients. Circuit simulations often do not reflect performance on biological media.

1 SPICE Simulators

There are many closed source variants of the SPICE circuit simulator freely available or under commercial licenses. Examples include PSPICE (Cadence), HSPICE (Synopsys), LTspice (Linear Technology), and TINA-Ti (Texas Instruments). Two open source variants are ngspice (GPL), and the original code for spice3f5 (BSD) [1]. There are a host of alternatives with an equally broad selection of slightly incompatible or ill-defined SPICE dialects. Various SPICE implementations support an array of control constructs, measurement and plotting which automate common tasks. Circuits designed with SPICE tools, with some careful simulation work, can achieve admirable correlation with lab-measured results. In general, simulation tools accelerate the debug/test loop by enabling rapid iteration and exploration which minimizes hardware design costs.

The circuit simulation tools are capable of large and small signal steady-state, transient, and noise performance simulations on linear elements (RLC), mixed signal (integrated digital logic), complex semiconductor device models, and behavioural models. Many semiconductor products, such as op-amps and high-speed interfaces, have vendor-supplied SPICE models for use in analog design and signal integrity work.

SPICE models support integer exponents, for example ideal capacitors are modelled in the frequency domain as $1/\mathbf{j}\omega C$. A second-order filter has a transfer function with a denominator term $(\mathbf{j}\omega)^2$. Fractional exponents do not generally occur in common circuit analysis practice. This mismatch in simulator capability means that when a circuit (for example, an EIT current source) is connected to biological media, instead of a resistor phantom, performance often degrades to an unanticipated level.

2 Fractional-Order Models

Biological media can be characterized by multiple "dispersions" over frequency using a summation of multiple Cole-Cole models as a complex-valued permittivity $\hat{\epsilon}$ [2, 3]

$$\hat{\epsilon} = \epsilon_\infty + \sum_m \frac{\Delta\epsilon_m}{1 + (\mathbf{j}\omega\tau_m)^{(1-\alpha_m)}} + \frac{\sigma}{\mathbf{j}\omega\epsilon_0}$$

for the permittivity of free space $\epsilon_0 = 8.8542 \times 10^{-12}$ F/m, angular frequency $\omega = 2\pi f$, and $\mathbf{j} = \sqrt{-1}$. The high frequency permittivity ϵ_∞, low-frequency ionic conductivity σ, time constants τ_m, breadth $0 \leq \alpha_m \leq 1$, and the permittivity difference $\Delta\epsilon_m = \epsilon_m - \epsilon_\infty$ for a particular permittivity ϵ_m at $\omega\tau_m \ll 1$ define a complex permittivity which can be converted to complex conductivity σ^*

$$\sigma^* = \mathbf{j}\omega\epsilon_0\hat{\epsilon} = \sigma + \mathbf{j}\omega\epsilon_0\epsilon \quad \text{for} \quad \hat{\epsilon} = \epsilon + \frac{\sigma}{\mathbf{j}\omega\epsilon_0}$$
$$= \sigma + \mathbf{j}\omega\epsilon_0 \left(\epsilon_\infty + \sum_m \frac{\Delta\epsilon_m}{1 + (\mathbf{j}\omega\tau_m)^{(1-\alpha_m)}} \right)$$

and used in an FEM forward EIT computation.

This FEM can be reduced to an n-port RLC network over n electrodes [4] and simulated in SPICE tools as a linear passive subcircuit to calculate large signal steady-state solutions at a fixed frequency ω. For broadband behaviour over frequency ω, the non-integer exponent α_m leads to *fractional-order models*.

Modelling of fractional-order super-capacitor, lithium-ion, and photo-voltaic power performance (off-grid, mobile and low power devices), certain control systems, chaotic and biological systems modelling, and the design of fractional-order filters and ultra low voltage sensors suffer from a lack of support in circuit simulation tools [5].

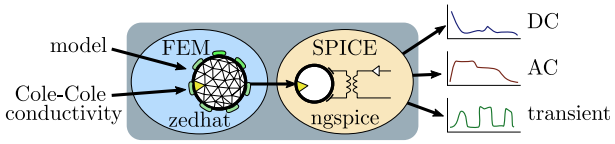

Figure 1: Proposed connection between EIT FEM modelling (zedhat) and SPICE circuit simulation (ngspice)

3 Solution Approaches

Common simulators do not have fractional-order elements (e.g. Constant Phase Elements) for building biological models. Two approaches have been suggested in recent publications: to fit RC networks to the complex impedance spectra [6] over a certain bandwidth, or to solve using nonlinear Modified Nodal Analysis (MNA) and convolution [7, 8].

We propose a code linking specialized EIT FEM software and SPICE circuit simulation (Figure 1). We invite discussion of the utility of such a tool building on an integration of FEM Cole-Cole models from the new `zedhat` software library.

References

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